Name _____ Test 1, Fall 2020

1) Multiply the two matrices below or state why they cannot be multiplied. (15 points)

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 0 & -3 & 1 \\ 2 & 2 & 4 \end{bmatrix}$$

2) Find the null space of the matrix below. (16 points)

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 2 & 6 \\ 0 & 0 & 4 \\ 0 & 0 & 7 \\ 0 & 0 & 0 \end{bmatrix}$$

3) Reduce the matrix below to reduced row echelon form. (16 points)

$$\begin{bmatrix} 1 & 2 & 5 & 6 \\ 0 & 2 & 4 & 6 \\ 2 & 4 & 10 & 15 \\ 1 & 4 & 9 & 12 \end{bmatrix}$$

4) Answer the questions below (3 points each)

(A) Let $A$ be a $2 \times 4$ matrix. How many solutions does $A\vec{x} = \vec{0}$ have?

(B) Let $A$ be a $4 \times 2$ matrix. In row reduced echelon form, it has 3 rows of zeroes. How many solutions does $A\vec{x} = \vec{0}$ have?

(C) Let $A$ be a $3 \times 3$ matrix such that $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ has one solution. How many solutions does $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$ have?

(D) If $A$ is a $7 \times 7$ matrix and the dimension of the row space is $5$, what is the dimension of the column space?

(E) If $A$ is a $7 \times 5$ matrix such that $A\vec{x} = [1 \quad 1 \quad 1 \quad 1 \quad 1]^T$ has infinitely many solutions, what is the maximum dimension of the row space of $A$?

5) Find the length of the vector below. (8 points)

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}$$

6) Are the vectors below orthogonal to each other? Justify your answer. (8 points)

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 4 \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ 3 \\ 5 \\ 0 \end{bmatrix}$$

7) Identify a good partition to use to multiply the matrices below, then multiply them. (7 points)

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ 0 & 0 & 4 & -5 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 9 & 11 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 6 \end{bmatrix}$$

8) Multiply the vector $\vec{v} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ by the scalar $5$. (8 points)

9) Add the matrices below. (7 points)

$$\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 4 \end{bmatrix}$$